**Fluid Mechanics**
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**Lecture - 14**
**Conservation of Momentum: Example Problems**

Welcome all of you for this course on fluid mechanics. In the last class we discussed about conservation of mass and the momentum and its applications. To continue to that conservation of mass and momentum and its applications, today I will deliver lecture on this topic and also I will solve some example problems to illustrate it how we can use conservation of mass and momentum equations to solve real life problems.

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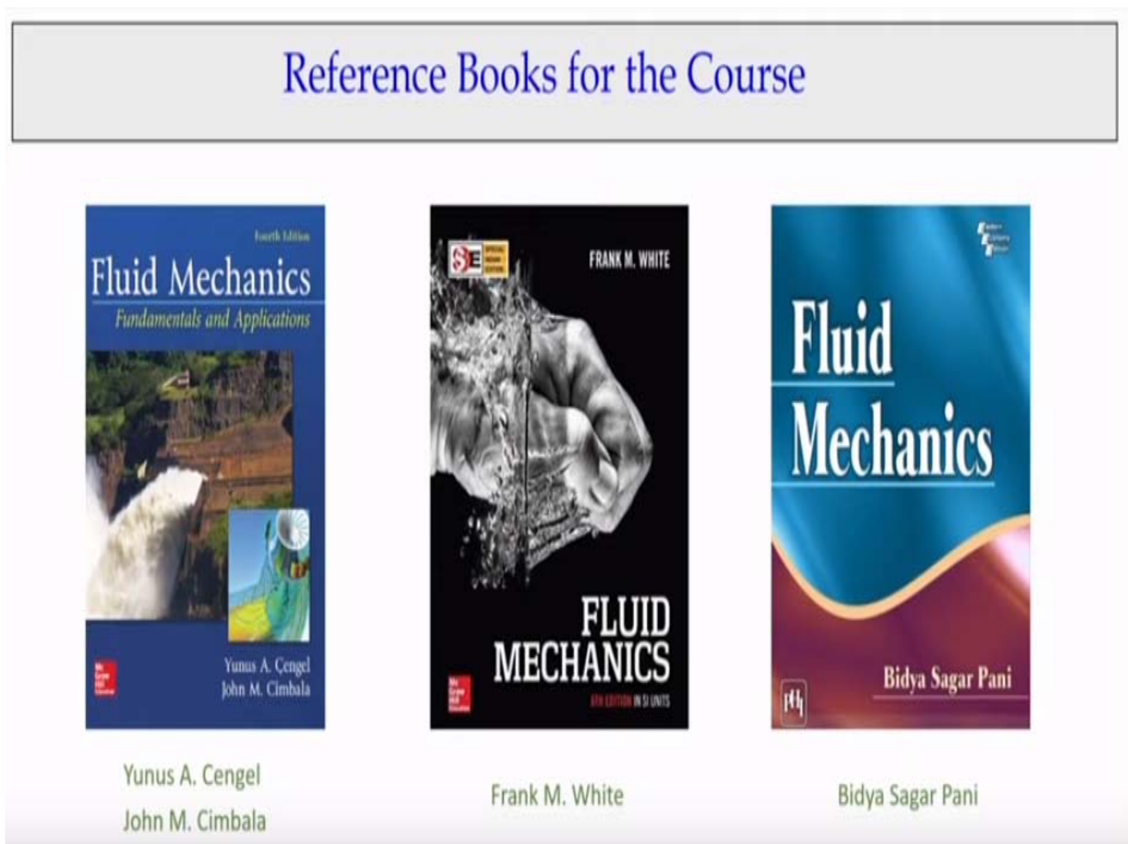


Again, I can talk about this that some of the examples the level of the fluid mechanics what I have been teaching it, its level of the fluid mechanics fundamental and applications, Cengel, Cimbala or close to the Fluid Mechanics by F.M. White.

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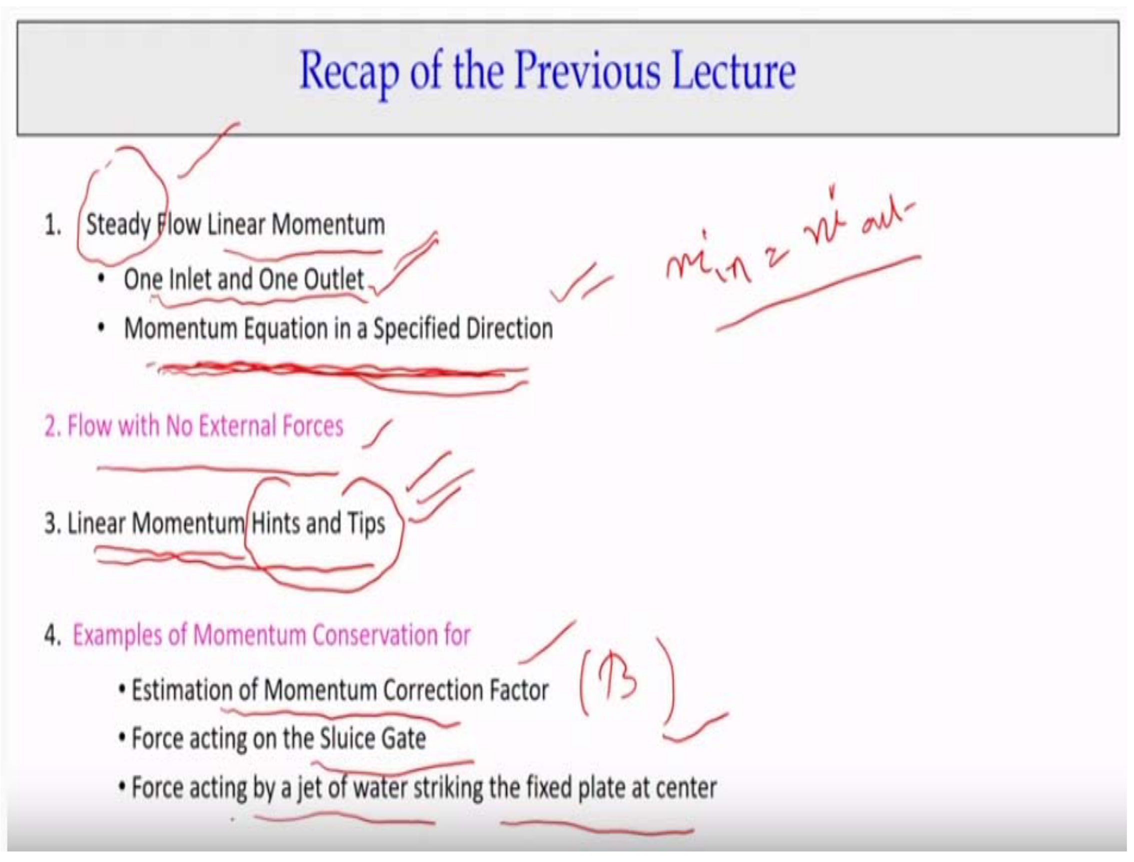


Let us come back to what we discussed in the previous class that we discussed that how we can approximate linear momentum equations from Reynolds transport theorems to a specific cases like one inlet, one outlet, which is very simplified problems, when you have a one inlet and one outlet. And second thing is that is you know it the momentum equation is vector equations.

$$m_{in} = m_{out}$$

But many of the case we can solve the problems in a specified directions or equating momentum equation in a specified direction. Then we can solve the problems. So we can align the specific axis rotation in such a way that using only one equation we can solve the problems. So these are the simplifications that the problems where we have one inlet and one outlet.

And the momentum equations we can apply for a specific directions. Most important qualities that we consider always is steady flow conditions. This is the approximations what we do it, the steady flow conditions where there is no change of the pressure or the velocity distributions with respect to the time. So it is a steady condition. We apply the steady linear momentum equations.

Again you simplified for the one inlet one outlet case, where the mass inflow will be equal to mass outflow. So this is very simplified case we will get it. Similar way, when I apply the momentum equation in specific directions that means, as you know it the momentum equations, we can write in three scalar components in x, y, z for Cartesian coordinate system.

So that way we can apply the momentum equations for the polar coordinate systems also. So if you look at that way, we can apply the momentum equations in a specific direction such a way that we can solve the problems. Flow with no external forces like a spacecraft, there is no external forces are there. In that case, what simplifications we do it.

Then we talk about when you apply this linear momentum equations, we should follow hints and the tips like we follow the free body diagrams in solid mechanics, whenever you apply the linear momentum equations, you have to draw control volume, the control surfaces. How to define this control volume and control surface whether it is a fixed control or movable control, how should be the control surface should be there.

That should be a major emphasis when you draw a control volume and the control surfaces and that hints we should consider it how these pressure distributions and velocity distributions we should assume it or we can have a proper assumption which is valid for that problems, that we should highlight it. In the last class we solved many problems like force acting on the Sluice gate, force acting a jet water striking the fixed plate at the centers.

Also we talk about the momentum corrections factors that means beta. How do you do you compute the beta value which is the momentum correction factors.

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Now let us come to today lectures what I will talk about that. We will discuss many things with a starting with a flow structures in hydraulic jump. Then again I will talk about or repeat the linear momentum hints and tips. That is what again I will repeat it. Again we will going to solve some of the previous GATE questions or example problems based on linear momentum and mass conservation equations.

Flow with no external forces, I will repeat that part. Then we will solve one examples, cases where no external forces is there specifically the spacecraft problems that what we will solve it and end of the day we will have a summary for these lectures which is more theoretical.

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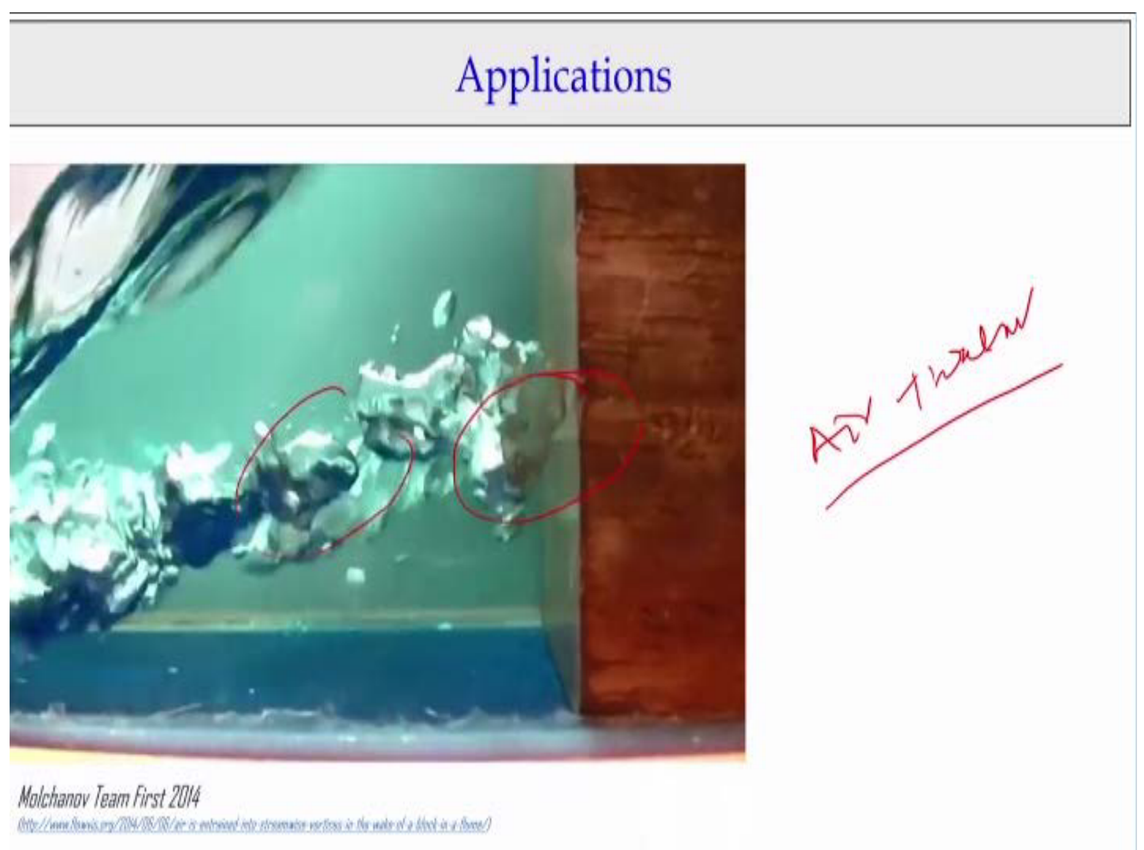


Now if you look, it is very interesting photographs what is there. It is available, you just type the flow visualizations. It is a great things are nowadays available if you just type the flow visualizations in any Google search engine you can see this type of flow visualization tools are available. If you look it what I am going to show from this photographs, it is clearly indicating that if I have flow jet and there is a mixing of air, there is mixing of air and the water okay, how the things are changing it.

If you look at that, the bubbles what is representing it here, those are all are the air bubbles. So and if this is the water jet, there is formations of water and air bubbles mixing with water. So interesting pictures are coming up and if you just visualize the flow, how it is very mixing of air and waters and that what change the flow systems.

So with this just a flow visualization tool let us come back to the our applications of linear momentum equations. Again, I am repeating this part of the tips of linear momentum applications, what are the tips are necessary to do that.

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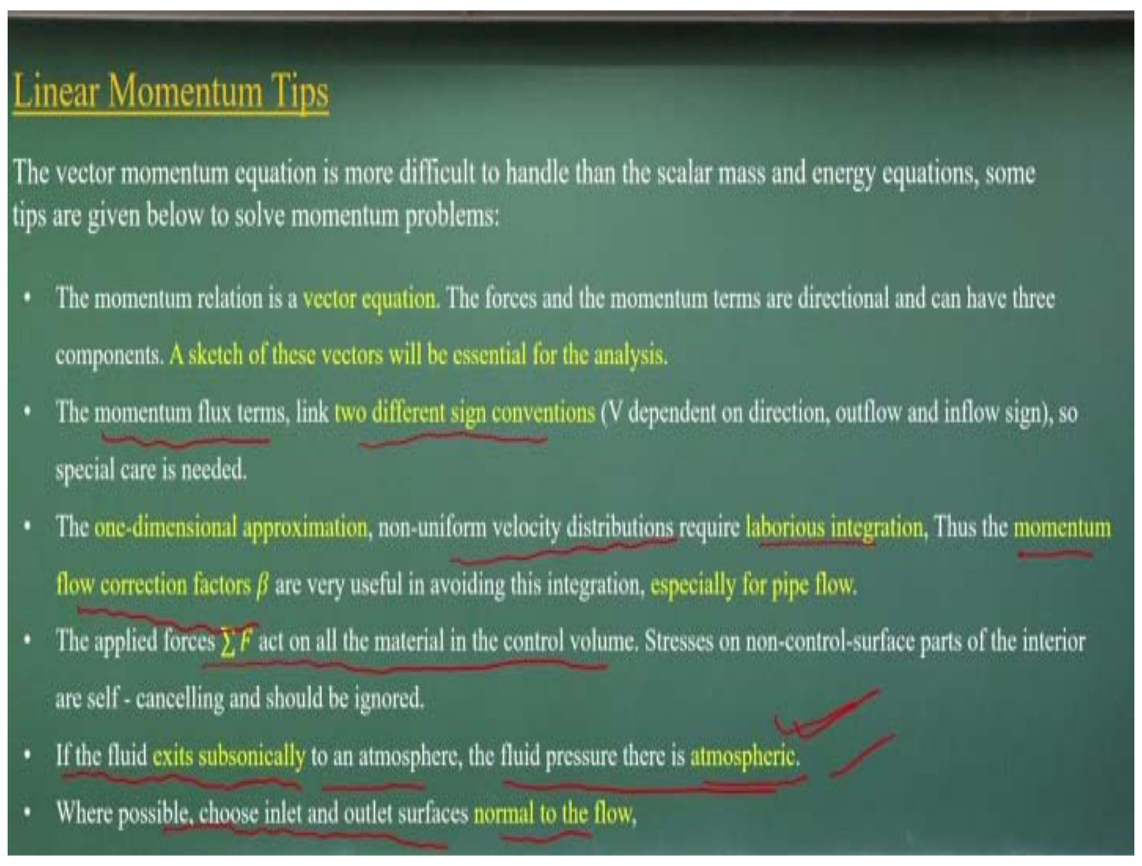


One thing you should remember it this momentum relationship what you get it is a vector equations. That means it has three scalar component, x component, y component and z component. We have the velocity vectors in the three component, V x, V y, V z. So similar way we have the force factors, which have three components. The momentum relationship what we have also the three components.

But many of the time we reserve it into the scalar components like writing the momentum occasions in x directions, y directions or the z directions, we do that. Second thing what I am to highlight is that momentum flux terms. The most of the times when you compute the momentum flux terms, we should look it what is the relationship, what is the angle between velocity vectors and the normal vector to the control surface.

If the scalar product of these two vectors is positive sign, that means it will have a positive flux is coming into the control volume. If it is a negative sign, it indicates it is going out from the control volume. So that is the scalar product of the velocity vectors and the normal vectors to the control surface. That what to consider. Always you scale the velocity vectors and the normal vectors then try to find out what is the angle between them.

Most of the times what you make it these two surface should have a such a way that the theta the angle between these two vectors should be zero or the pi, okay 180 degree. So it will be easy for us to do vector product. So that way we get either a positive or the negative sign convections. And most of the times if you look it the flow are not uniform velocity distributions. It is non uniform velocity distributions.

Because of that, we should have the momentum flow correction factor beta because of the velocity distributions is not uniform. But many of the time we simplified it and say that the beta is equal to the 1. We consider it that the flow distribution is uniform and we do the or we solve the problems.

But if you try to understand it sometimes we can use the beta value, the momentum flow correction factors when it will be much larger than beta equal to 1. We can use that for especially for the pipe flow. When beta is close to 1, it may help us indirectly not to consider the velocity distributions. But when beta is more than the 1 we should consider the beta values to find out approximated how much the momentum clocks going through these control surface.

Similar way when you apply this control volume concept, within the control volumes we do not talk about that as it can be considered is a trace field and that can self canceling each others. The mostly we would consider the control volume gross characteristics. Within the control volume we do not consider it how the flow variations are there, the pressure variations and velocity variations or the density variations we do not consider that inside the control volume.

And this is what very good it is not atmospheric is a quite valid conditions when you have the fluid exits to an atmospheric and the fluid flow is subsonic. Then we can assume it the fluid pressures at that locations is atmospheric pressures. These assumptions is quite valid. So many of the times we use whenever the flow jets are going to the atmospheres and we anticipated is that the flow is subsonic.

That means the flow Mach number is less than 1. In that case, we can have the fluid pressures we can assume it or it is quite valid the atmospheric pressures. And the to appropriate inlet outlet surface, generally we take it flow normal to that. The otherwise we can solve the problems using the scalar product of the velocity and the normal vectors. It is possible to do it, but it will be laborious to do the integration part or it takes time to solve the problems.

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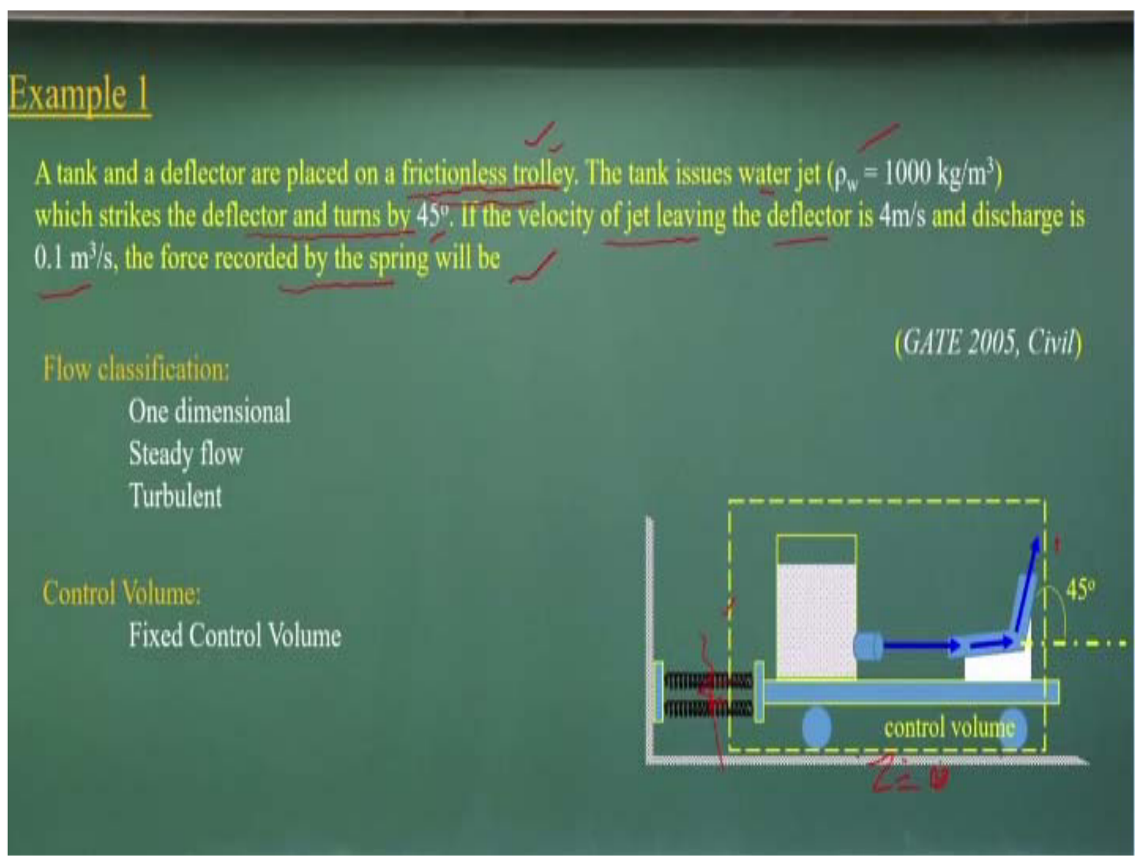


Now, let us come it to solve example one which is GATE 2005 civil engineering problems. The problem is that there is a tank or deflectors are placed in a frictionless trolley. Okay, there is no frictions components, okay. The water issues the water jet which is have a the density of the water is 1000 kg per meter cube. It strike to a deflectors, turn 45 degree.

[A tank and a deflector are placed on a frictionless trolley. The tank issues water jet ($\rho_w$ = 1000 kg/m$^3$)
which strikes the deflector and turns by 45°. If the velocity of jet leaving the deflector is 4m/s and discharge is 0.1 m$^3$/s, the force recorded by the spring will be]

If velocity of jet leaving the deflector is 4 meter per second and discharge is equal 0.1 meter cube per second, what could be the force record by the spring, that is the problem. So if you look at that that is what we have sketched it here. There is a tank which is having the waters and getting jet of the waters from this tank and there is a frictionless trolley that means there is no frictional force acting on this.

The problem is quite simplified for us. And there is a deflector which is making a 45 degrees to turn these things. We need to compute it, how much force is going to act on the spring. That is the problem. It is very easy problem, we can solve it. But let us follow very systematic approach what I discussed in the last class, we will go step by step. First step is that to classify the problems.

Flow classification:

One dimensional

Steady flow

Turbulent

These problems is one dimensional flow and steady flow because the discharge what is coming out from this jet can consider is a steady. Flow will be the turbulent and consider a fixed control volume okay. That is what the control volumes I have consider it. So the force what is acting here that is a component here. Other the locations if you look at this water jet is going out from this.

Other locations we can take it the pressure equal to the atmospheric pressures. And here the shear stress is equal to zero because frictionless trolley. So there is no friction force is acting on this. So it will becomes zero. So there is no force component is here due to the friction. So water jet that component will come it. That is what the water jet is coming out from this. So we have chosen the control volume, fixed control volume. Our object is to compute what is the force recorded by the spring.

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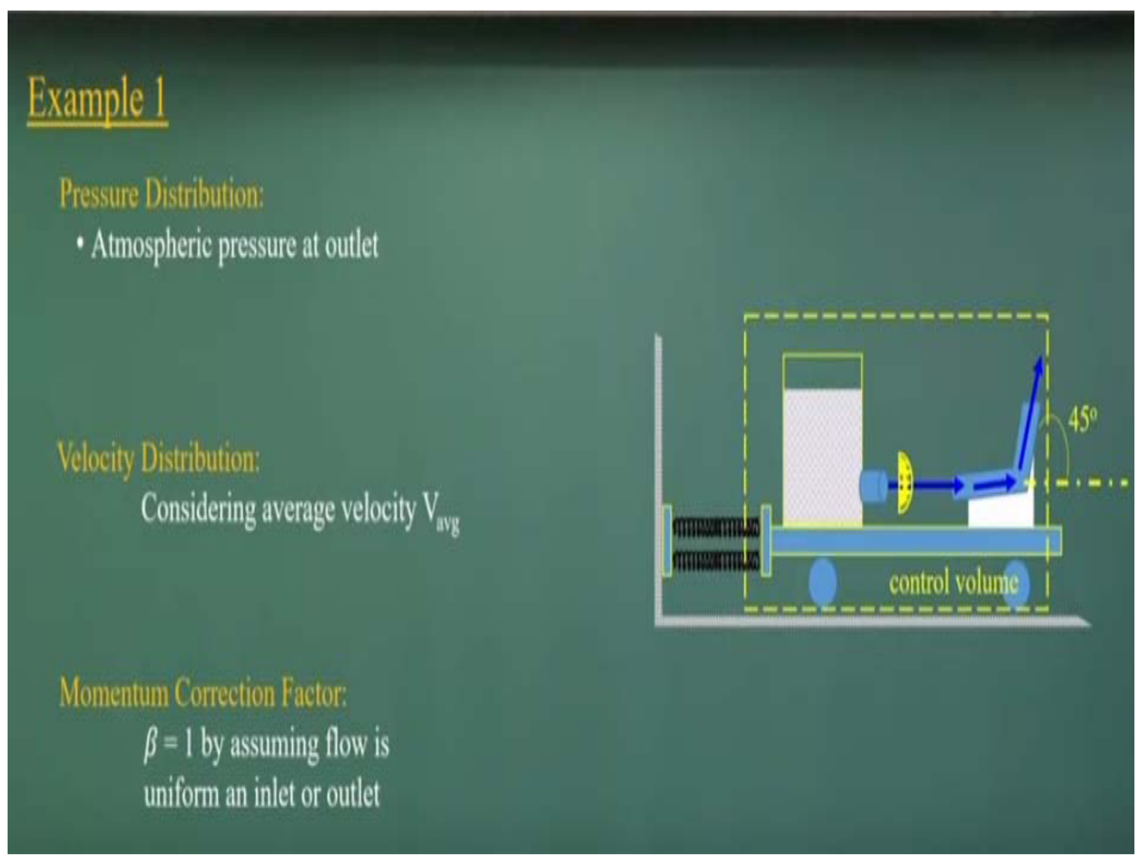


So the pressure distributions at the outlet will be the atmospheric pressure. Here also we have consider the velocity distributions again average velocity. No doubt there will be a velocity distributions will come it here. After deflectors also velocity distribution will come it, but we here assume it is uniform velocity and the beta will be the one value for this.

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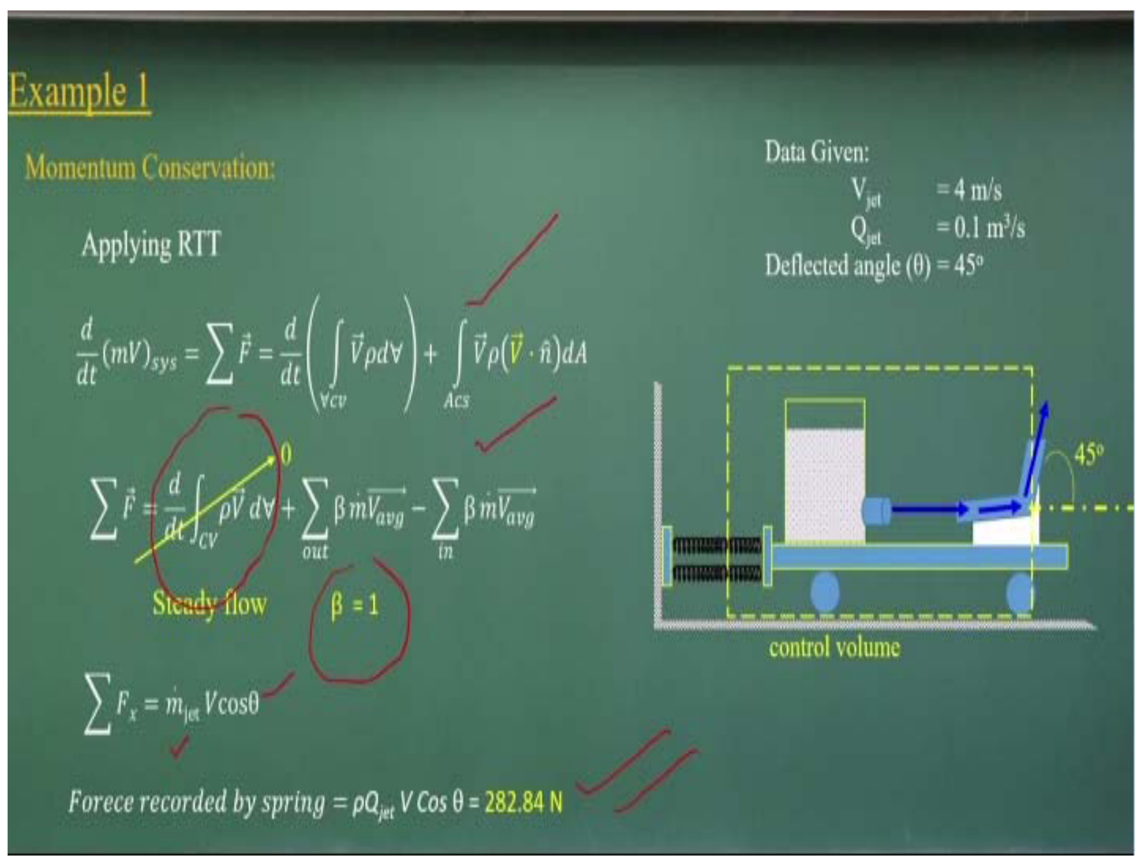


Now let us apply the directly the momentum conservation equations here. If I apply this momentum conservation equation this some of the forces should equal to the rate of the change of momentum flux within the control volume is equal to the net outflux of momentum flux passing through these control surface.

Data Given:

$V_{jet}$ = 4 m/s

$Q_{jet}$ = 0.1 $m^3/s$

Deflected angle (θ) = 45°

Momentum Conservation:

Applying RTT

$$\frac{d}{dt}(mV)_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{\forall cv} \vec{V}\rho d\forall\right) + \int_{Acs} \vec{V}\rho(\vec{V}\cdot\hat{n})dA$$

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall + \sum_{out} \beta\, \dot{m}\overrightarrow{V_{avg}} - \sum_{in} \beta\, \dot{m}\overrightarrow{V_{avg}}$$

Steady flow, $\frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall = 0$

$$\sum Fx = \dot{m}\text{jet}\, V\cos\theta$$

$Forece\ recorded\ by\ spring = \rho Q_{jet}\ V\ Cos\ \theta$ = 282.84 N

You can look it the cos theta component will come it as the x directions and force recorded by spring if I compute what will be the mass, $\rho$ into Q. Q object which known to us, V is known to us, $\theta$ is known to us, we can compute what will be the force acting in terms of Newton.

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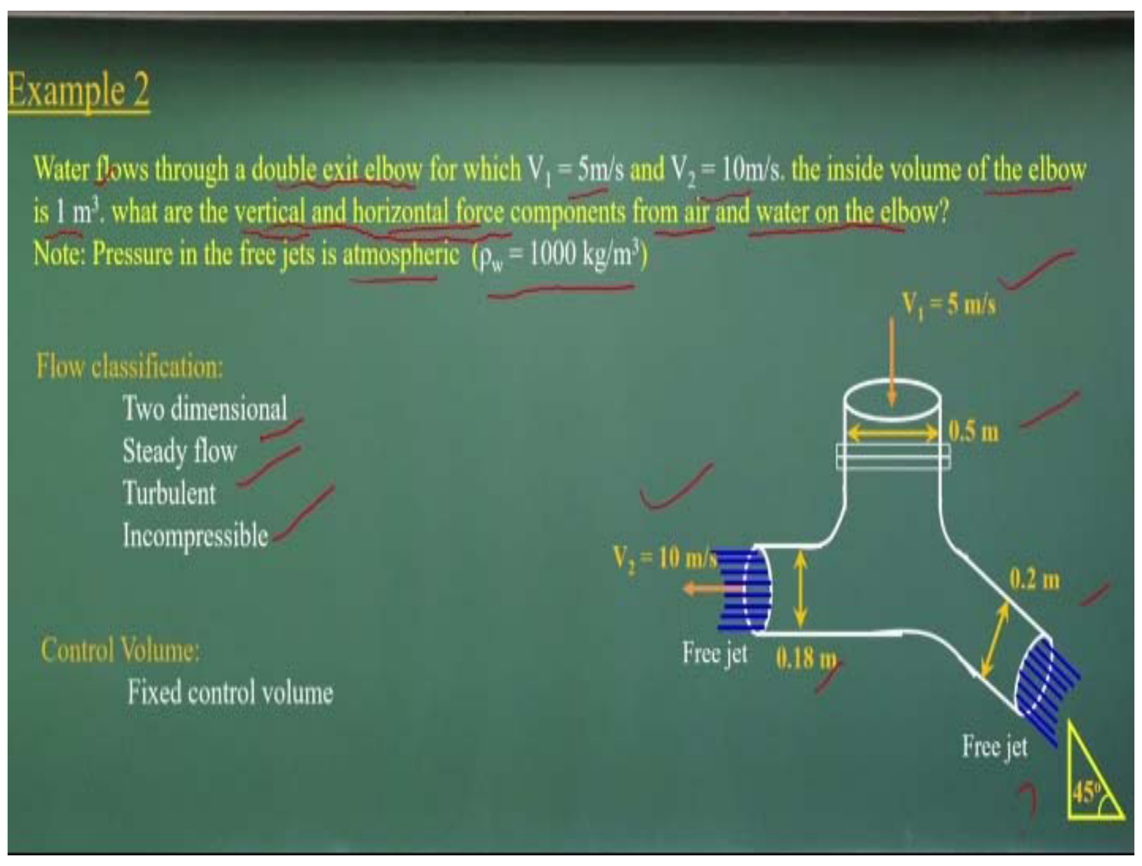


Now let us go to the second example 2 where we have the water flows through a double exit elbow. Okay this is elbow type of concept, where we have a V 1 velocity and V 2 velocity. One is 5 meter per second another is 10 meter per second. Inside the volume of the elbow is 1 meter cube, okay the volume of this. What are the vertical and horizontal force component of from air and water on this elbow if pressure in the free jet is atmospheric and the unit weight of the water will be 1000 kg per meter cube.

[Water flows through a double exit elbow for which $V_1$ = 5m/s and $V_2$ = 10m/s. the inside volume of the elbow is 1 $m^3$. what are the vertical and horizontal force components from air and water on the elbow?
Note: Pressure in the free jets is atmospheric ($\rho_w$ = 1000 kg/$m^3$)]

Now if you look it the sketch of the problems, there are the free jet in the two part. And there is a velocity here, the velocity here, but the velocity is unknown here. We do not know the velocity of this part. There is a diameter of this elbow part. The first we have to apply the mass conservation equations. Then we will apply linear momentum equations to compute what is the vertical, horizontal force components.

Flow classification:

Two dimensional

Steady flow

Turbulent

Incompressible

Is very simple things that here we will apply mass conservations equations. Then we will apply two momentum equations, one for the vertical directions and other is horizontal directions to get what is the vertical force component, what is the horizontal force component. First let us define the problem. Problem is two dimensional.

As you seen it that here the flows are coming, there is not change of the flow volume within this, this. So we can consider is a steady problem, turbulent, incompressible. That what we can consider as a flow classification. We can consider is a fixed control volume like this. This is what will be the control volumes. We have the surface like this. We have a surface like this.

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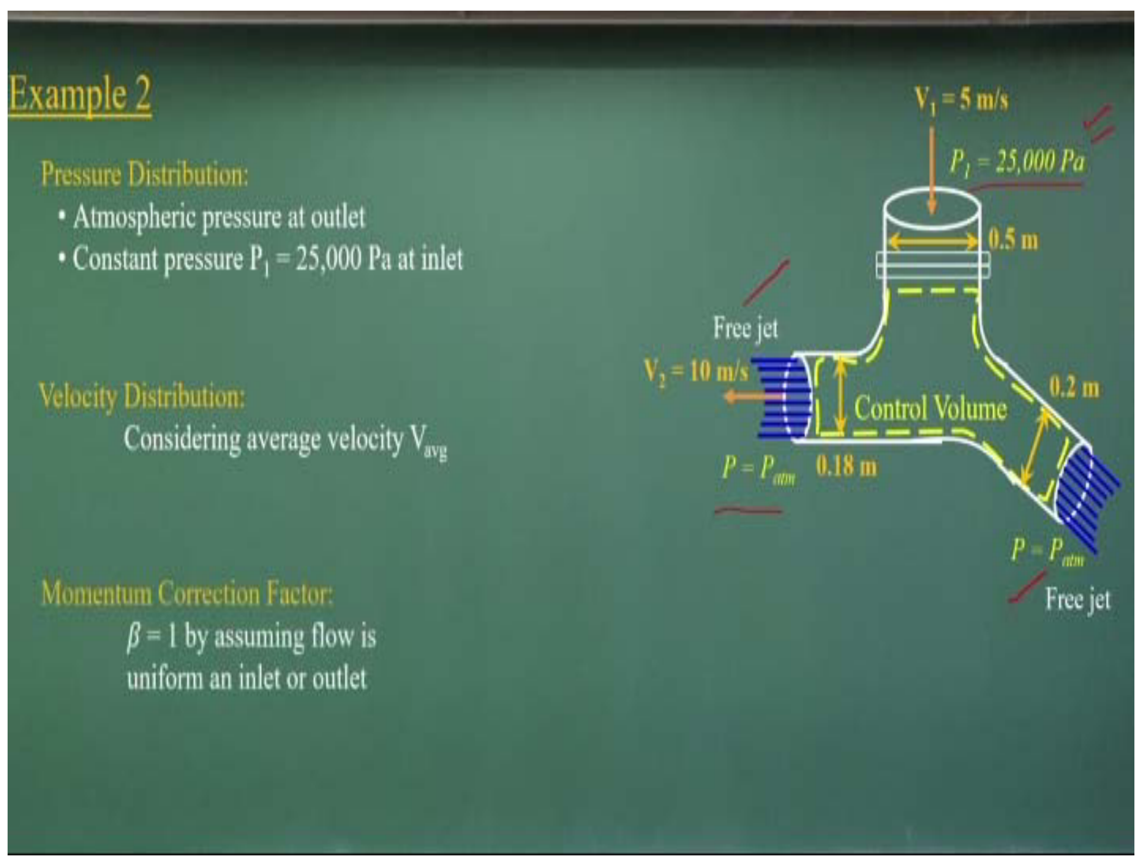


So if we have this fixed control volumes, first we will find out that the pressures. The pressures at the inlet is given is 25,000 Pascal. At the outlet is pressure is atmospheric pressure. That what is there. So you have the free jet at these two locations where pressure is the atmospheric pressures. At these two point you have a pressure equal to atmospheric pressure, but at this point I have the pressure which is 25,000 Pascal, Newton per millimeter square.

So if you have that and we use a concept of average velocities. We neglect the velocity distributions. We use the beta equal to the 1 as for the inlet and the outlet.

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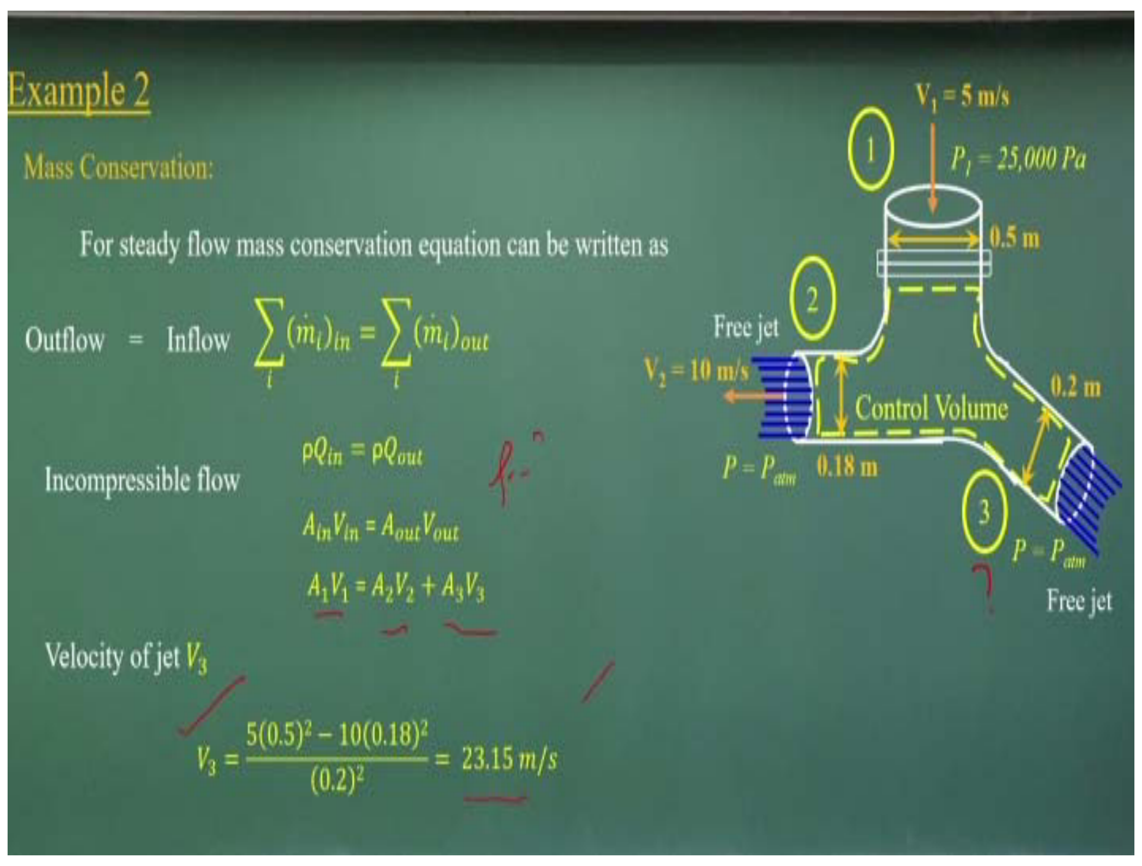


Then we will apply mass conservation equations that let us apply mass conservation equations to compute it what could be the velocity at the section 3. So we are applying the mass conservation equations for these control volumes that to compute what could be the velocity at this point. So if I apply this mass conservation equation which are very simple form is rate of mass influx into the control volume is equal to rate of the mass outflux from this control volume.

For steady flow mass conservation equation can be written as

Outflow = Inflow

$$\sum_i (\dot{m}_i)_{in} = \sum_i (\dot{m}_i)_{out}$$

$\rho Q_{in} = \rho Q_{out}$

$A_{in}V_{in} = A_{out}V_{out}$

$A_1V_1 = A_2V_2 + A_3V_3$

Velocity of jet $V_3$

$$V_3 = \frac{5(0.5)2 - 10(0.18)2}{(0.2)2} = 23.15\ m/s$$

Simple substituting the area and the velocity of $V_1$, $V_2$ and you can compute it the velocity of z passing through the section 3 will be 23.15 m/s.

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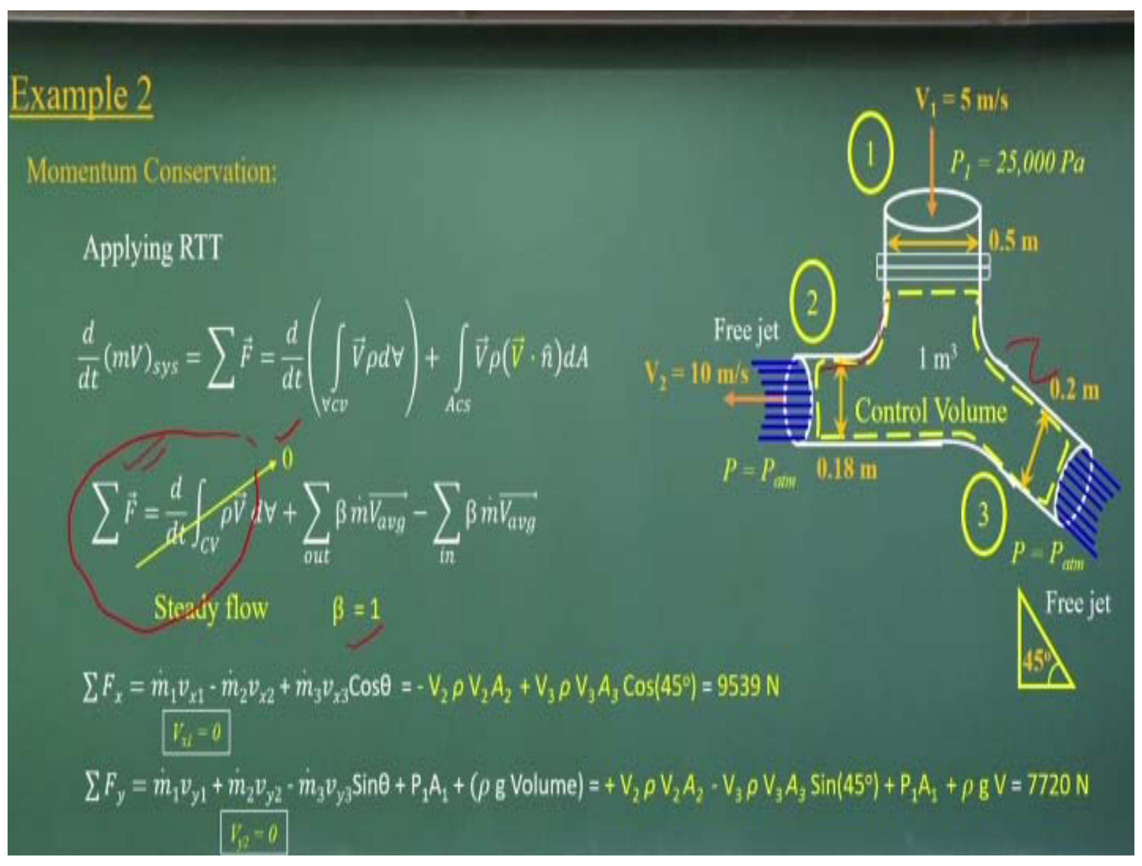


So if I apply this momentum conservation equations for this control volume, when I apply this momentum conservation equations, the first things what I writing the control volume equations, Reynolds transport theorems and then I am simplifying it. So again it is a steady problem, okay. And I look it the momentum flux in and out assuming the beta is equal to the 1. And our objective is now to compute the force components.

We have the consider the control volume inside this control volume, you can think that there will be a frictional resistance from the surface of the 1. But that part we are neglecting it as compared to the force component what is acting it. So this way we can neglect the frictional force which is there near the wall, that part we are neglecting it to compute it what will be the force component.

Applying RTT

$$\frac{d}{dt}(mV)_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{\forall cv} \vec{V}\rho d\forall\right) + \int_{Acs} \vec{V}\rho(\vec{V}\cdot\hat{n})dA$$

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall + \sum_{out} \beta\, \dot{m}\overrightarrow{V_{avg}} - \sum_{in} \beta\, \dot{m}\overrightarrow{V_{avg}}$$

Steady flow, $\frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall = 0$, $\beta = 1$

$\sum Fx = \dot{m}1vx1 - \dot{m}2vx2 + \dot{m}3vx3 Cos\theta = - V_2 \rho V_2 A_2 + V_3 \rho V_3 A_3 Cos(45^o) = 9539$N

$V_{x1} = 0$

$\sum Fy = \dot{m}1vy1 + \dot{m}2vy2 - \dot{m}3vy3 Sin\theta + P_1A_1 + (\rho$ g Volume$) = + V_2 \rho V_2 A_2 - V_3 \rho V_3 A_3 Sin(45^o) + P_1A_1 + \rho$ g V = 7720 N

$V_{y2} = 0$

So if I apply this part, then F x will come it to the 9539 Newton. Similar way the force component acting on this y direction that one will be the 7720 Newton.

The approximations and all the velocity component, zero component if you look it and we have resolved the force, momentum flux component as a cos theta and sin theta. We have also considered the force due to the pressures at this point, which is and also we consider the weight of the fluid here. If you look at this, the weight of the fluid is considered in the y direction, that components are there.

The force due to the pressure one also we have consider it and these are all momentum flux component that what total what is come in these parts. So there is a force F x direction and F y direction. We noted that the force due to the pressure at this point we have computed which is can consider as a gauge pressure, it is not the absolute pressure.

And also we have considered the weight component of rho g into the volume. Combining the all the terms we got it the force acting the y direction will be 7720 Newton.

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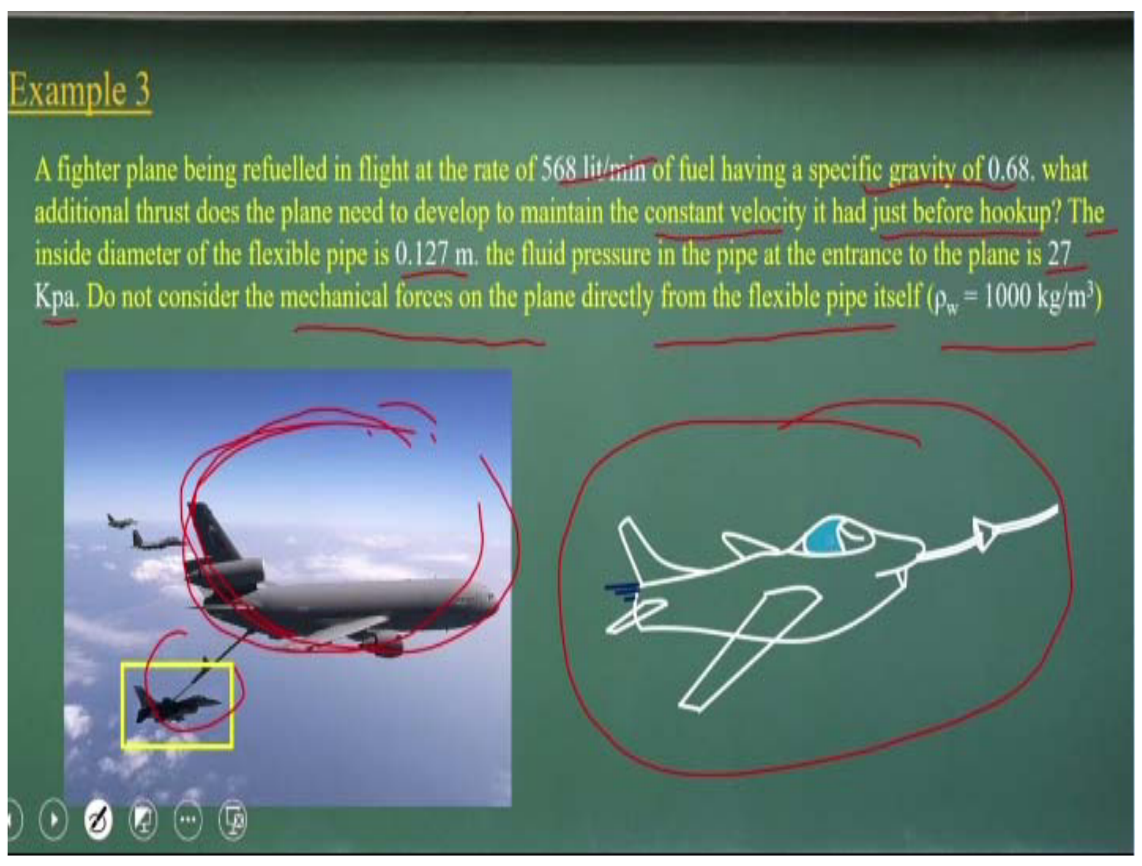


Let us take another example. A fighter plane being refueled in a flight at the rate of 568 liter per minute of fuel having specific gravity of 0.68. What additional thrust does the plane need to develop or maintain constant velocity it had just before hookup. The inside diameter of the flexible pipe is 0.127 meters. The fluid pressure in the pipe at the entrance to the plane is 27 kilo Pascal.

[A fighter plane being refuelled in flight at the rate of 568 lit/min of fuel having a specific gravity of 0.68. what additional thrust does the plane need to develop to maintain the constant velocity it had just before hookup? The inside diameter of the flexible pipe is 0.127 m. the fluid pressure in the pipe at the entrance to the plane is 27 Kpa. Do not consider the mechanical forces on the plane directly from the flexible pipe itself ($\rho_w$ = 1000 kg/m$^3$)]

And if you neglect the mechanical forces on the plane directly to the flexible pipe and you consider the unit weight of water will be 1000 kg per meter cube. The problem what graphically it is shown it in this figure, this is the main aircraft and this is the what the fuelling aircraft okay which is fuelling this the aircraft from this sides. That is what new technologies what is develop it.

In air itself we can refuel the aircraft from this part. So if it is that is the conditions, what could be the additional thrust does the plane need to develop to maintain the constant velocity between these two. They should move it the same velocity. They should move it with the same velocity so that the refueling can be done. That is the problem. Now we try to look it what is the additional thrust is necessary.

The problem is now simplified in this part that there is aircraft like this and it is hooking it, the fuel hooking to this one.

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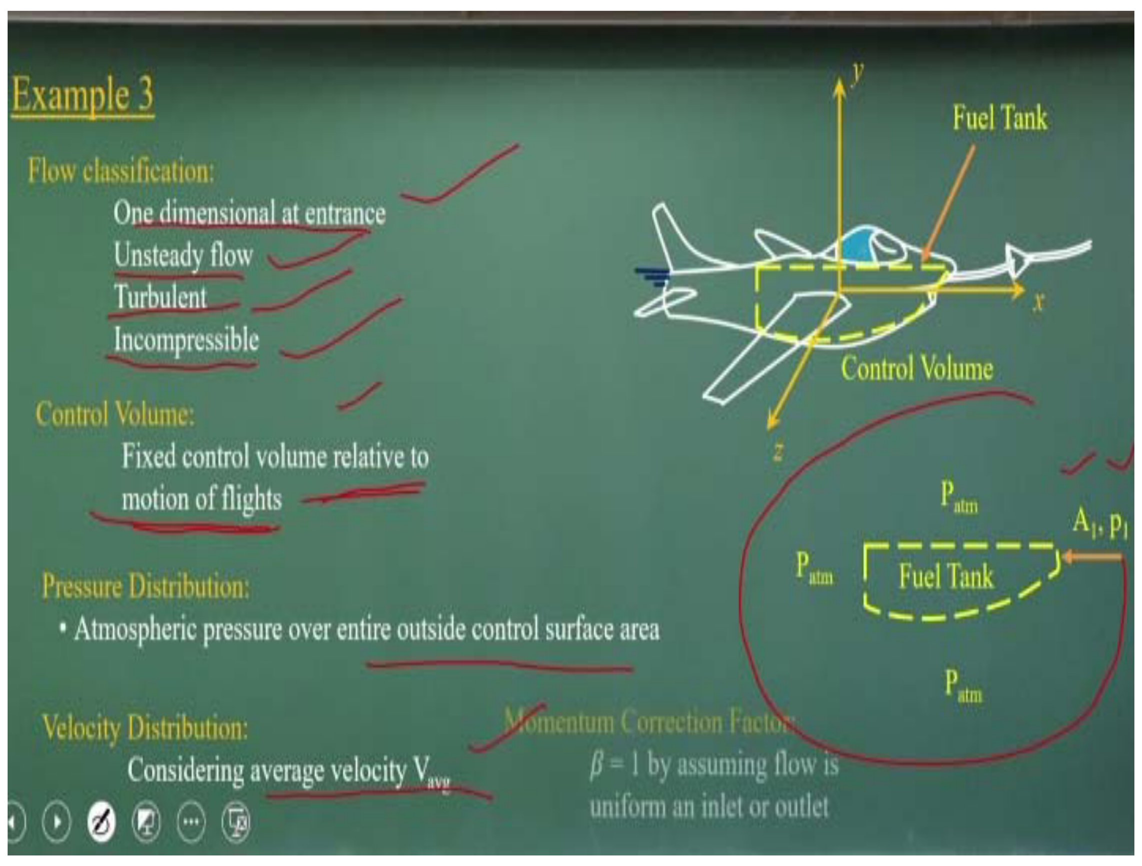
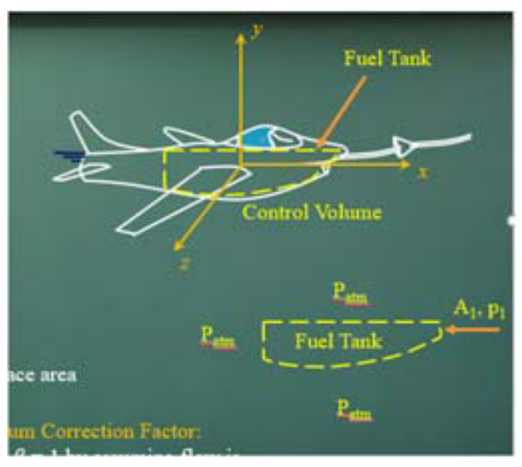


And if I define the fuel tank and this coordinate of x, y, z the problems can be considered is one dimensional. Fluid is unsteady flow, but we can simplify it make it the steady flow; turbulent, incompressible. So these three conditions will be there; one dimensional, we are not considering the velocity distributions and all. The unsteady, turbulent and incompressible flow. That is what the classifications.




Flow classification:

One dimensional at entrance

Unsteady flow

Turbulent

Incompressible

Here it is movable control volume, but since we have the two flights are moving it with respect to the refueling aircraft we can consider is a fixed control volume, relative motions to the flight okay. So that the basic assumptions of these type of problems is a fixed control volume which is related to the motion of the flights. If that is the conditions, the pressure distributions enter outside the control surface.

We can consider is atmospheric pressure what will be there like this the problems now, everywhere you have atmospheric pressure and the pressure, the fuel what is injecting it, it is P 1 pressure having area A 1. The everywhere the control surface can be considered is P equal to the atmosphere pressure. The velocity distributions again we can take is average velocity distributions V average value, beta equal to the 1.

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